Modelling and analysis of intermodal food grain transportation under hub disruption towards sustainability

Lohithaksha M Maiyar*, Jitesh J Thakkar*
*Department of Industrial & Systems Engineering, Indian Institute of Technology Kharagpur, Kharagpur, West Bengal, India-721302

Abstract

Escalating global food security concerns across several nations has shifted the focus of policy makers towards risk adaptive sustainable food grain operations. This paper builds a sustainable food grain transportation model for intermodal transportation operations between two Indian states, in the presence of hub disruption. A hub and spoke system is used to connect origin and destination warehouses through intermodal hubs in a multi-layered network. The problem is formulated as a multi-period mixed integer nonlinear single objective optimization problem considering minimization of transportation, hub location, rerouting, environmental and social costs with near optimal shipment quantities and hub allocations as the prime decisions. The proposed MINLP is solved using Particle Swarm Optimization with Differential Evolution (PSODE), a superior metaheuristic to deal with NP-hard problems. Convergence graphs and global optimal costs are reported for small, medium and large size instances consisting of 1824, 9768 and 28848 variables respectively, inspired from food grain industry in the southern part of India. Pareto plots are generated to capture the complementarity between economical and socio-environmental cost categories for all instances. The effect of hub location, hub disruption, cost consolidation and vehicle resource availability factors on individual and total costs is studied through sensitivity analysis. Results indicate that food grain demand is fulfilled with 14% increase in the mean total cost for single hub disruption case and with 40% increase for multiple hub disruption. Finally,
managerial implications provide specific factor level recommendations for different strategic objectives.

Keywords: Intermodal transportation; food grain shipment; disruption; particle swarm optimization with differential evolution; Sustainability

1. Introduction

Increasing food grain procurement patterns, global food security concerns and the unquenched demand for staple food steer the need for realizing efficient food grain management from procurement to consumption posing numerous challenges to scientists from operations research, decision sciences, and systems engineering community. In developing countries, food grain wastage is a prime cause of food shortage to targeted population. Approximately, 33% of the edible parts of food produced for human intake, gets wasted globally, which amounts to 1.3 billion ton per annum (http://www.fao.org/save-food/resources/keyfindings/en/). Food loss levels observed in industrialized countries are almost as same as losses observed in developing countries. The difference being that in developing countries more than 40% of the produce is wasted in initial stages of the food supply chain where as in industrialized or developed countries approximately same amount of wastage is observed at consumer and retailer levels (http://www.fao.org/save-food/resources/keyfindings/en/). As a result, alleviating rising food security concerns is a challenge in present day context, especially with expanding population. This paper delves into intermodal transportation intricacies and attempts to model interstate transportation issues under disruption for Indian Food Grain Supply Chain (FGSC), the second largest in the world.

According to Song et al., (2014), transportation planning plays a pivotal role in determining overall costs in any supply chain. Especially, for geographically wide spread procurements end-to-end delivery of staple food involves transport through multiple modes, heterogeneous resources, and
adherence to time restrictions. Significant wastages are observed at intermodal transfer points, manual loading/unloading points and different intermediate stages of the supply chain (CAG Report) due to poor infrastructure and inefficient resource planning. Rail freight transport, in India, serves majority of the sectors with total freight shipment of over 1.107 billion tons and approximately 50 million tons of food grains (Indian Railways annual report, 2011-2012). Food Corporation of India (FCI) is a nodal government organization dedicated to manage food supply chain operations across the country. Food grain losses are mainly attributed to improper planning, infrastructure malfunctioning, and inefficient resource utilization. Further, unexpected disruption of an intermodal hub or warehouse facility would severely hamper the customer service level, food security and the economy. The aftermath of floods at West Bengal, India, 2016 and earthquake at Nepal, 2015 witnessed irreparable losses to food grain storage facilities depriving local population of the basic amenity. In such cases, enabling adaptive capacity and diverting the transportation operations through alternative hubs is a constructive approach to reduce delays and losses (Williams et al., 2017). Food grains are staple food to India and neighboring countries, and thus, by streamlining the food grain transportation in the presence of disruption with simultaneous focus to carbon emission reduction and internalization of social cost, this paper strongly relates to economic, environmental and social aspects of sustainable logistics.

This work exploits the four stage categorization of Indian food grain shipment framework conceptualized by Maiyar et al. (2015). The first stage involves intra-state food grain allocation within surplus or deficit state. At the end of this stage, food grain surplus states retain the excess stock to satisfy the demand of food grain deficit states in the second stage. The second stage involves interstate transfer of food grains in between central level storage facilities using multiple modes of transport. The third and fourth stages constitute intra-state distribution from central level
warehouses to block level warehouses and fair price shops. This paper particularly addresses the second stage while focusing on sustainable freight transportation in intermodal context. An interstate transportation model is built on a hub and spoke framework with the peripheral nodes representing the central level FCI warehouses and the hub nodes representing intermodal hubs. Connections between non hub nodes (spokes) represent road transport and hub to hub connections represent rail mode of transport. The transportation is allowed to be realized in single or multiple time periods as per the availability of vehicle resources and intermodal hub capacity. Here, the transportation cost function considers consolidation factor for intermodal transfers. The key decisions are to determine the spatial and temporal distribution of food grain shipment quantities and origin/destination hubs to be located in the presence of hub disruption. The contributions from this work are three fold. Firstly, a sustainable nonlinear mathematical model representing the Indian FGSC is formulated considering the minimization of transportation, hub location, rerouting, environmental, and social costs simultaneously under hub disruption. The proposed model accommodates for multiple route conditions and adheres to hub restrictions, emergency hub constraints, flow balance equations, and vehicle capacity restrictions. Secondly, a superior hybrid meta-heuristic, PSODE is tailored and employed to solve the proposed MINLP. Thirdly, sensitivity analysis is carried out to understand the behavior of transportation, hub location, rerouting, environmental, social and total shipment costs under the influence of changing cost consolidation factor, vehicle resource availability, hub location and hub disruption levels. Pareto analysis is conducted to visualize the complementarity between total shipment cost and socio-environmental costs. To the best of our knowledge, this effort is a novel contribution to the domain of intermodal transportation and hub location problems in FGSC context fostering economic, ecological and social sustainability.
The remainder of the paper is structured as follows. Section 2 outlines the relevant literature. Section 3 describes the problem with illustrative figures. Section 4 presents the proposed mathematical model for Indian FGSC with detailed description of objective function and constraints. Section 5 discusses the solution methodology and its adaptation to current context. Section 6 presents plan of experiments for computational validation and sensitivity analysis. Later section 7 explains the results and insights developed through sensitivity analysis. Finally, section 8 summarizes the paper with conclusion and future work.

2. Literature review

Disruption management in the context of intermodal transportation continues to exist as part of transportation research and of particular interest to policy makers since the 1990s. Azad et al. (2016) propose an optimization-based methodology considering the trade-off between cost of mitigation strategy and the expected cost of disruption in a railroad network. In the literature, many studies have found focus on intermodal transportation problems (Barnhart and Ratliff, 1993; Boardman et al., 1997; Hokey, 1991; Macharis and Bontekoning, 2004; Southworth and Peterson, 2000). However, these studies highlight the intermodal transportation planning of transportation activities without much focus towards sustainability aspects and disruptions. In the presence of environmental disruptions, Kamalahmadi and Parast (2017) analyzed different risk mitigation strategies to curb the effect of their impact on supply chain wide costs. In the recent past, risk and sustainability have been strategically addressed. (Brusset and Teller, 2017). Beermann, (2011) emphasized on the importance of resilience thinking in business perspective for changing climate adaption. Williams et al. (2017) described the implementation of adaptive capacity in transportation networks during disruptions and highlighted the importance of disruption aware design of sustainable supply chains.
In the recent last decade, sustainability is often addressed in the triple bottom line (TBL) perspective focusing on economic, environmental and social objectives (Ahi and Searcy, 2015). Though, empirical research has significantly dealt with sustainability implementation in the TBL perspective, analytical models are yet to be well addressed (Brandenburg et al., 2014). Nevertheless, combination of economic and environmental objectives are more profound in the analytical literature (Hassini et al., 2012). Dey, (2006) has pointed out importance of design for failure as a key factor to inculcate sustainability in the ecological dimension. Further, design for integration of two or more individual operational objectives has also been considered as sustainable practice in the economic dimension. Lejeune, (2006) proposed an integrated inventory-distribution model to address sustainability in this perspective. Incorporating social sustainability is still a challenge both individually and in the TBL perspective. Brandenburg et al., (2014) emphasize that the amount of customer and employee satisfaction achieved determines the degree of social sustainability in supply chain. Abreu and Camarinha-Matos, (2008) propose a value identification system concerning the employees in the collaborative organization context. However, customer ended social sustainability is yet to be incorporated into modelling and is open domain for research. In this paper, the proposed model gains social importance by internalizing the socio-economic impact of non-financial factors (noise pollution, accidents and congestion) while designing sustainable freight transportation in the presence of disruption.

Facility location decisions and network design problems have been rigorously dealt in OR literature, both individually and in combination with one another. In a pioneering contribution, Minoux (1989) put forth efficient network models and solution methodologies. Balakrishnan et al. (2004) provided survivable network design solutions using split connections. An integrated approach was proposed by Melkote and Daskin, (2001) to deal with facility location and network
design decisions for a transportation problem. Studies have been conducted to optimize location of rail or road terminals for freight transport (Arnold et al., 2004) and to develop an integrated model for the evaluation of road-rail intermodal freight hub locations (Sirikijpanichkul and Ferreira, 2006). Focusing on reliability aspects, Cui et al. (2016) and Li and Ouyang, (2010) studied facility location design problems under disruption, while Peng et al. (2011) emphasized on reliable logistic network design under facility disruption. However, studies that capture benefits of consolidation in the presence disruption while evaluating shipment quantity and shipment route for multiple time periods are limited in literature. This study intends to bridge this gap in the domain of food grain transportation.

Hub and spoke formulations have been studied and implemented in airline, shipping and railway sectors to manage traffic congestion, to reap consolidation benefits and to stream line goods transfer operations at intermodal points (Bai et al., 2014; Macharis and Bontekoning, 2004). Intermodal hub-and-spoke network design problem with multiple stakeholders and multi-type containers was dealt by Meng and Wang (2011). In the presence of disruption, Azizi et al. (2016) proposed a mathematic model with a hub-and-spoke system for single mode and single time period scenario. Parvaresh et al. (2014) modelled a hub and spoke network as a bi-level leader-follower based multi-objective problem with p-hub constraints. They captured monetary advantage of flow consolidation at hubs by considering discount factor or cost consolidation factor for shipments routed through hub to hub links. Sustainable transportation, high fuel efficiency, product safety and consistent deliveries are identified as benefits of integrating intermodal transportation with hub and spoke system (Bouchery and Fransoo, 2015).

In the recent past, transportation research in general has marked its appearance into food grain domain, especially for rice and wheat. However, the focus of intermodal transportation issues in
the presence of disruption for food grain context is limited. A clean review of production and
distribution operations pertaining to agri-foods is seen in Ahumada and Villalobos, (2009). Asgari
et al. (2013) proposed a transportation and storage model to facilitate wheat movement in Iran.
Focusing on silo operations, Mogale et al. (2017) developed a MINLP model adhering to seasonal
procurement, scientific storage restrictions, varying demand, transportation mode and vehicle
capacity constraints in Indian context. Hyland et al. (2016) highlighted the importance of shuttle
train service in domestic grain supply chain considering trucking, elevator storage, and rail
transportation. A multi-objective optimization model was proposed and solved by Thakur et al.
(2010) to analyze the tradeoff between grain lot sizes and cost of blending in grain handling
operations. In the context of risk mitigation, Ge et al. (2016), developed a hybrid optimization-
simulation model to study the Canadian wheat supply chain. A game theory based robust grain
supply chain design considering post-harvest losses was developed by An and Ouyang, (2016).

Critical examination of the extant literature (Table 1) reveals that there is a need for a sustainable
and integrated multi-period model to support intermodal operation to evaluate transportation and
hub location decisions in the presence of disruptions. The social and environmental dimension
associated with food grain operations especially in India intensify the need for such a modelling
approach. In line with the above concern, the underpinning contribution of this paper lies in
formulating a MINLP model to simultaneously minimize transportation, hub location and
rerouting, environmental and social costs in the presence of hub disruption for Indian FGSC. The
environmental cost is calculated by taxing the total carbon emissions released from road and rail
transport. The social cost captures the social cost of carbon emissions, noise pollution, accidents
and congestions. The model incorporates emergency hub constraints in addition to several real
time constraints relevant to multi-period intermodal transportation. A well-known swarm
intelligence technique, PSODE is tailored and employed to solve the proposed problem. Furthermore, sensitivity analysis is carried out to figure out the effect of cost consolidation factor, vehicle resource availability, hub location and hub disruption levels on individual and total costs.
Table 1: Comparative study of relevant literature with present work

<table>
<thead>
<tr>
<th>Study</th>
<th>TRC</th>
<th>HC</th>
<th>RC</th>
<th>EC</th>
<th>SC</th>
<th>Emergency hub constraint</th>
<th>Noise pollution</th>
<th>p-hubs constraint</th>
<th>Intermodal/multimodal</th>
<th>Vehicle/ hub capacity</th>
<th>Multi period</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parvaresh et al. (2014)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>MILP*</td>
</tr>
<tr>
<td>Ghaffarinasab and Motallebzadeh, (2018)</td>
<td>✓</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>x</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>MILP</td>
</tr>
<tr>
<td>Suh and Ryerson, (2017)</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>x</td>
<td>NL*</td>
</tr>
<tr>
<td>Hyland et al. (2016)</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>x</td>
<td>L*</td>
</tr>
<tr>
<td>Azad et al. (2016)</td>
<td>✓</td>
<td>x</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>x</td>
<td>ILP*</td>
</tr>
<tr>
<td>Ishfaq and Sox (2011)</td>
<td>✓</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>x</td>
<td>✓</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>ILP</td>
</tr>
<tr>
<td>Liotta et al. (2015)</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>x</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>✓</td>
<td>x</td>
<td>MILP</td>
</tr>
<tr>
<td>An and Ouyang (2016)</td>
<td>✓</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>MINLP*</td>
</tr>
<tr>
<td>Mogale et al. (2017)</td>
<td>✓</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>MINLP</td>
</tr>
<tr>
<td>Asgari et al. (2013)</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>ILP</td>
</tr>
<tr>
<td>Present study</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>MINLP</td>
</tr>
</tbody>
</table>

L- Linear, NL- Non-linear, ILP- Integer linear problem, MILP- Mixed integer linear problem, MINLP- mixed integer nonlinear problem
3. Problem description

The Indian FGSC is categorized into four stages: (1) Intra-state transportation (2) Inter-state transportation (3) Intra-state distribution up to block level and (4) Intra-state distribution from block level to fair price shops. As discussed earlier, this paper attempts to particularly address the inter-state transportation stage among the four stages. In doing so, the intermodal hub and spoke problem developed in this paper, mainly involves two players: Food Corporation of India (FCI) and the Indian Railways. FCI is responsible for the portion of transportation occurring through road and the Indian Railways is responsible for the hub – hub rail hauls. The intermodal transfer points are owned by the FCI and are capped with maximum hub transfer capacity restrictions. In our problem, the origin and destination states are divided into finite number of regions and each region has finite number of warehouses. The set of potential intermodal hubs is a subset of set of warehouses. The food grain shipment is carried out between origin warehouses in the surplus state to the destination warehouses in the deficit state through the selected intermodal hubs. A diagrammatic illustration of food grain movement in the hub and spoke system is provided in Fig. 1.

![Diagram](image)

**Fig. 1.** Network configuration of warehouses and hubs in hub and spoke system

The unselected potential intermodal hubs act as peripheral nodes (supply or demand nodes) in the hub and spoke system. At the origin intermodal hubs, food grains are unloaded from the trucks and loaded into trains while at the destination intermodal hubs they are unloaded from trains and
loaded into trucks. As mentioned earlier, during the disruption of an origin or destination hub, the loss of demand is satisfied by rerouting the food grains through emergency hubs. Fig. 2 gives a clear view of the route network of the intermodal operation through emergency hubs.

While satisfying the demand of food grains for Indian FGSC at minimized total shipment costs the paper attempts to address the following questions:

- What is the optimal combination of origin-destination (o-d) pair of warehouses?
- How much quantity is to be transported for each o-d pair and through which hub?
- What is the optimal hub location plan?
- How will the sudden failure of origin and destination nodes affect the supply network costs?

Fig. 2. Intermodal network with emergency hubs

4. Mathematical model
Generally, intermodal problems with simultaneous hub location decision are closely associated to p-hub median allocation problem (Ishfaq and Sox, 2011). Parvaresh et al. (2014) studied the problem in the presence interdictions and referred to it as p-hub median problem with intentional disruption (PHMI). The mathematical model developed in the current paper is an extension of PHMI formulation. It is important to mention that since the p-hub median problem is a proved NP-hard problem (Alumur and Kara, 2008), the extended formulation is also NP hard. Sustainable factors can be included in the modelling of transportation and hub location networks across different dimensions. In the context of this problem, economic sustainability is incorporated at the objective function level in the form of rerouting costs. With respect to the modelling carried out in this paper, it is important to highlight the following assumptions:

- Single food grain commodity is transported
- Demand is deterministic in nature
- Vehicles carry Full Truck Load (FTL) transport
- At least one warehouse is present in each region
- At least one hub is open in each state
- Seasonal variations are ignored
- Emergency hubs are not disrupted
- The shipment is realized in single time period

In this formulation, separate indices are defined for regions and warehouses of origin and destination states as shown in Table 2. The problem is defined with the help of eight independent finite sets (Table 3). The set of potential hub locations is a strategic decision input and is taken as a subset of set of warehouses in the respective origin or destination state.

Table 2. Table of Indices
Table 3. Table of Sets

<table>
<thead>
<tr>
<th>Set</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_o )</td>
<td>Set of regions in origin state ( o )</td>
</tr>
<tr>
<td>( R_d )</td>
<td>Set of regions in destination state ( d )</td>
</tr>
<tr>
<td>( W_p )</td>
<td>Set of FCI warehouses in origin region ( p )</td>
</tr>
<tr>
<td>( W_o )</td>
<td>Set of FCI warehouse in origin state, ( W_o = \bigcup_{p=1}^{[R_o]} W_p )</td>
</tr>
<tr>
<td>( W_q )</td>
<td>Set of FCI warehouses in destination region ( q )</td>
</tr>
<tr>
<td>( W_d )</td>
<td>Set of FCI warehouses in destination state, ( W_d = \bigcup_{q=1}^{[R_d]} W_q )</td>
</tr>
<tr>
<td>( H_o )</td>
<td>Set of potential hub locations in origin state ( o ), ( H_o = \bigcup_{p=1}^{[R_o]} W_p )</td>
</tr>
<tr>
<td>( H_d )</td>
<td>Set of potential hub locations in destination state ( d ), ( H_d = \bigcup_{q=1}^{[R_d]} W_q )</td>
</tr>
<tr>
<td>( Z )</td>
<td>Set of route conditions (road/rail)</td>
</tr>
<tr>
<td>( T )</td>
<td>Set of time periods</td>
</tr>
</tbody>
</table>

In the literature, benefit of flow consolidation at the intermodal hubs is quantitatively captured through cost consolidation factor \((\alpha | (0 < \alpha < 1))\). Given that, the unconsolidated unit transportation cost through intermodal hubs \( k \ (k \in H_o) \) and \( m \ (m \in H_d) \) in time period \( t \ (t \in T) \)
and route condition $\xi \ (\xi \in Z)$, is $TR_{km\xi t}$, the actual hub to hub unit transportation cost is calculated as, $\alpha(TR_{km\xi t})$, where, $1-\alpha$ represents the fractional reduction in the unconsolidated price ($TR_{km\xi t}$), by virtue of scale economies in consolidation.

$$CT_{ikmj}^{\xi t} = RO_{ik\xi t} + \alpha TR_{km\xi t} + RD_{mj\xi t}, \forall i, \forall j, \forall k, \forall m, \forall \xi, \forall t$$ (1)

A linear cost function as shown in Eq. (1) is used to calculate intermodal transportation cost for single unit of food grain from warehouse $i \ (i \in W_o)$ to warehouse $j \ (j \in W_d)$ in time period $t \ (t \in T)$ and route condition $\xi \ (\xi \in Z)$. Here, the unit transportation cost, $CT_{ikmj}^{\xi t}$ is realized as the summation of three components. First, the unit cost of transportation by road from warehouse $i \ (i \in W_o)$ to origin hub $k \ (k \in H_o)$ in time period $t \ (t \in T)$ and route condition $\xi \ (\xi \in Z)$, represented by $RO_{ik\xi t}$, second, the unit hub-hub transportation cost ($\alpha TR_{km\xi t}$), and third, the unit cost of transportation by road from destination hub $m$ to warehouse $j \ (j \in W_d)$ in time period $t \ (t \in T)$ and route condition $\xi \ (\xi \in Z)$, represented by $RD_{mj\xi t}$.

The problem consists of the following six types of decision variables and aims to capture the shipment quantity, shipment route, hubs located and the hubs disrupted across origin and destination states:

$$x_{ikmj}^{\xi t} \quad \text{Quantity of food grain flow from origin warehouse } i \text{ to destination warehouse } j \text{ of through intermodal hubs } k \text{ and } m \text{ having route condition } \xi \text{ in time period } t \ , \text{ where,}$$

$$i \in W_d \ , \ j \in W_d \ , \ k \in H_o \ , \ m \in H_d \ , \ \xi \in Z \ , \ t \in T$$

$$y_{ikmj}^{\xi t} = 1, \text{ if there is flow from origin warehouse } i \text{ to destination warehouse } j \text{ through origin hub } k \text{ and destination hub } m \text{ in time period } t \ , \text{ where,}$$

$$i \in W_o \ , \ j \in W_d \ , \ k \in H_o \ , \ m \in H_d \ , \ \xi \in Z \ , \ t \in T \ , \ 0 \text{ otherwise.}$$
\[ z_{kt} = 1, \text{ if hub } k \text{ is open in time period } t, \text{ 0 otherwise} \]

\[ w_{mt} = 1, \text{ if hub } m \text{ is open in time period } t, \text{ 0 otherwise} \]

\[ \gamma_{kt} = 1, \text{ if hub } k \text{ is disrupted in time period } t, \text{ 0 otherwise} \]

\[ \delta_{mt} = 1, \text{ if hub } m \text{ is disrupted in time period } t, \text{ 0 otherwise} \]

Minimize \( TC = TRC + HC + RC + EC + SC \)  \hspace{1cm} (2)

\[ TRC = \sum_{t,z,k,m,j} (1 - \gamma_{kt})(1 - \delta_{mt})CT_{ikmj}^z x_{ikmj}^t \]  \hspace{1cm} (3)

The objective function (Eq. (2)) minimizes the total costs (TC) and is calculated, with the decision variables defined as above, as the summation of following **five major cost components**: (1) Transportation cost (TRC) (2) Hub location cost (HC), (3) Rerouting cost (RC), (4) Environmental cost (EC), and (5) Social cost (SC). The fourth and fifth components in the total cost equation are included to capture the purview of environmental and social sustainability simultaneously while minimizing total shipment costs. The first term of the objective function, TRC (Eq. (3)), aggregates the transportation cost from individual shipments being transported across all the routes passing through non-disrupted origin and destination hubs. The multiplication \((1 - \gamma_{kt})(1 - \delta_{mt})\) ensures that the aggregated cost excludes the cost of transporting disrupted shipments.

\[ HC = HC_o + HC_d, \]  \hspace{1cm} (4)

\[ HC_o = \sum_{k,j} F_k z_{kt} \]  \hspace{1cm} (5)

\[ HC_d = \sum_{m,j} F_m w_{mt} \]  \hspace{1cm} (6)

The second term of the objective function, HC, is calculated according to Eq. (4), where \( HC_o \) and \( HC_d \) are the total hub location costs at the origin and destination states respectively and are
evaluated as shown in Eq. (5) and (6), where, $F_k$ and $F_m$, are the fixed costs of opening the hubs at $k \ (k \in H_o)$ and $m \ (m \in H_d)$ respectively.

$$RC = RC_o + RC_d + RC_{od}$$  \hspace{5cm} (7)

$$RC_o = \sum_{t, \xi, i, k, m, j} \gamma_{kt}(1-\delta_{mt})C_{iemj}^x \xi_{kmj}^x$$  \hspace{5cm} (8)

$$RC_d = \sum_{t, \xi, i, k, m, j} (1-\gamma_{kt})\delta_{mt}C_{ikfj}^x \xi_{kmj}^x$$  \hspace{5cm} (9)

$$RC_{od} = \sum_{t, \xi, i, k, m, j} \gamma_{kt}\delta_{mt}C_{iefj}^x \xi_{kmj}^x$$  \hspace{5cm} (10)

The third term of the objective function, $RC$, is estimated based on the occurrence of one of the following three cases. First, disruption at only origin hub, second, disruption at only destination hub, and third, disruption at both origin and destination hubs. The rerouting cost associated with first, second and third cases is obtained from Eq. (8), (9), and (10) respectively, where $C_{iemj}^x$, $C_{ikfj}^x$, and $C_{iefj}^x$ are the corresponding unit transportation charges for routing the flow through the origin emergency hub, $e$ and, destination emergency hub, $f$, as appropriate. Due to the difference in the nature of routing, the above three cases are independent and mutually exclusive. Hence, for a general case, the rerouting cost, $RC$ is calculated by simple summation of the three costs, as shown in Eq. (7).

$$E = E_\phi(1-\gamma_{kt}, 1-\delta_{mt}, k, m) + E_\phi(\gamma_{kt}, \delta_{mt}, e, f) + E_\phi(1-\gamma_{kt}, 1-\delta_{mt}, k, m) + E_\phi(\gamma_{kt}, 1-\delta_{mt}, e, m) + E_\phi(1-\gamma_{kt}, \delta_{mt}, k, f) + E_\phi(\gamma_{kt}, \delta_{mt}, e, f)$$  \hspace{5cm} (11)

where, $E_\phi(K, L, k', m') = \sum_{t, \xi} \left( \xi_{\phi} + \xi_{\phi} \cdot \left( \sum_{i, k} KA_{i, k, \phi} \left[ \frac{\sum_{m, j} x_{kmj}^x}{V_\phi} \right] + \sum_{m, j} LA_{m, j, \phi} \left[ \frac{\sum_{i, k} x_{kmj}^x}{V_\phi} \right] \right) \right)$  \hspace{5cm} (12a)
and, \[ E_\psi(K, L, k', m') = \sum_{t, \xi, k, m} \left( \xi_{\psi} + \xi_{\psi}' \right) K L A_{k, m} x_{k, m} \]
\[ \sum_{t, l, j} x_{l, m}^\eta \right) \times \frac{V_{\psi}}{V_{\psi}} \]
\[ \forall(K, L) \in \{0,1\}, \forall k' \in H_\psi \cup \{e\}, \forall m' \in H_\psi \cup \{f\} \]

The fourth term in Eq. (2), \( EC \), represents the total environmental costs incurred and is estimated as \( EC = C_r E \), where \( E \) is the total emissions (Eq. (11)). \( C_r \) represents the price of carbon tax expressed in rupees per tonne of \( CO_2 \) released. \( E_\phi(K, L, k', m') \) (Eq. (12a)) and \( E_\psi(K, L, k', m') \) (Eq. (12b)) calculate the total \( CO_2 \) emissions as a function of binary variables, \( K \) and \( L \), and intermodal hub indices, \( k' \) and \( m' \), for road and rail transport respectively defined to collectively capture disruption scenarios inbetween origin and destination states. \( K, L, k' \) and \( m' \) take values as shown in Eq. (11) for respective scenarios of disruption as discussed earlier. \( \xi_{\psi} \) and \( \xi_{\psi}' \) are full and empty load \( CO_2 \) emissions (g\( CO_2 \)) respectively for transport by road in condition \( \xi \) (\( \xi \in Z \)), whereas \( \xi_{\psi} \) and \( \xi_{\psi}' \) hold similar meanings for rail transport. \( V_\phi \) and \( V_\psi \) are the capacities of a single truck and rake respectively. \( A_{ik} \), \( A_{mj} \), and \( A_{km} \) are geographical distances for intermodal linkages as defined in Table 4.

\[ SC = C_r E + \left( C_{n\phi} + C_{c\phi} \right) \left[ \Phi_1(1 - \gamma_{kt}, 1 - \delta_{mt}, k, m) + \Phi_1(\gamma_{kt}, \delta_{mt}, e, f) \right] \]
\[ + C_{a\phi} \left[ \Phi_2(1 - \gamma_{kt}, 1 - \delta_{mt}, k, m) + \Phi_2(\gamma_{kt}, \delta_{mt}, e, f) + C_{c\psi} \left[ \Phi_3(1 - \gamma_{kt}, 1 - \delta_{mt}, k, m) \right] \right] \]
\[ + \Phi_3(\gamma_{kt}, 1 - \delta_{mt}, e, m) + \Phi_3(1 - \gamma_{kt}, \delta_{mt}, k, f) + \Phi_3(\gamma_{kt}, \delta_{mt}, e, f) \right] \]
\[ \Phi_1(K, L, k', m') = \sum_{t, \xi, i, k} \left( 2K A_{ik} \left[ \frac{\sum_{m, l} x_{l, m}^\eta}{V_\phi} \right] \right) + \sum_{t, \xi, m, j} \left( 2L A_{mj} \left[ \frac{\sum_{i, k} x_{l, m}^\eta}{V_\phi} \right] \right) \]
\( \Phi_2(K, L, k^t, m^t) = \left[ \sum_{i, j, k, m} \left( K A_{ik} \phi \sum_{m, j} x_{ikmj}^{2t} \right) + \sum_{i, j, m, j, k} \left( L A_{m, j} \phi \sum_{i, k} x_{ikmj}^{2t} \right) \right] \) (15)

\( \Phi_3(K, L, k^t, m^t) = \sum_{i, j, m, j, k, m} K L A_{km, j} \left( \frac{\sum_{i, j} x_{ikmj}^{2t}}{V_{jr}} \right) \) (16)

The final term in the objective function (Eq. (2)), SC, captures the total accumulated internalized social costs. SC is evaluated using Eq. (13) - (16), where Eq. (13) aggregates four individual components of social cost. While the first component quantizes the social cost of carbon emissions as a function of total emissions evaluated using Eq. (12), the second and third components quantify the economic impact of noise pollution, congestion, and accidents for road transport using non-linear estimation functions \( \Phi_1 \) (Eq. (14)) and \( \Phi_2 \) (Eq. (15)) which represent total vehicle kilometers and ton-kilometers travelled. Similarly, the fourth component of SC measures the total social cost of negative externalities from total vehicle-kilometers travelled for rail transport using \( \Phi_3 \) (Eq. (16)). The list of parameters required for understanding remainder of the formulation carried out in this paper are described in Table 4.

\( x_{ikmj}^{2t} \leq My_{ikmj}^{2t}, \forall i, \forall k, \forall m, \forall j, \forall \xi, \forall t \) (17)

\( \sum_{i, j, m, j} y_{ikmj}^{2t} \leq (|Z|||W_o||H_d||W_d|) z_{kl}, \forall k, \forall t \) (18)

\( \sum_{i, j, k, j} y_{ikmj}^{2t} \leq (|Z|||W_o||H_d||W_d|) w_{ml}, \forall m, \forall t \) (19)

\( \gamma_{kl} \leq z_{kl}, \forall k, \forall t \) (20)
Table 4. List of problem parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_{jt} )</td>
<td>Demand for food grains at warehouse ( i ) region ( q ) in time period ( t, j \in W_d, t \in T )</td>
</tr>
<tr>
<td>( I_{it} )</td>
<td>Food grain inventory at warehouse ( i ) of region ( p ) observed in time period ( t, i \in W_o, t \in T )</td>
</tr>
<tr>
<td>( P_{it} )</td>
<td>Food grain procurement at warehouse ( i ) of region ( p ) observed in time period ( t, i \in W_o, t \in T )</td>
</tr>
<tr>
<td>( U_k )</td>
<td>Intermodal handling capacity of hub ( k, k \in H_o )</td>
</tr>
<tr>
<td>( U_m )</td>
<td>Intermodal handling capacity of hub ( m, m \in H_d )</td>
</tr>
<tr>
<td>( n_o )</td>
<td>Number of origin hubs allowed to be disrupted in any time period</td>
</tr>
<tr>
<td>( n_d )</td>
<td>Number of destination hubs allowed to be disrupted in any time period</td>
</tr>
<tr>
<td>( \lambda_{it} )</td>
<td>Number of trucks available in warehouse ( i ) of region ( p ) in time period ( t, i \in W_o, t \in T )</td>
</tr>
<tr>
<td>( \mu_{kt} )</td>
<td>Number of rakes available in origin hub ( k ) in time period ( t, k \in H_o, t \in T )</td>
</tr>
<tr>
<td>( \rho_{mt} )</td>
<td>Number of trucks available in destination hub ( m ) in time period ( t, m \in H_d, t \in T )</td>
</tr>
<tr>
<td>( A_{ik\phi} )</td>
<td>Geographical distance by road from warehouse ( i ) of region ( p ) to intermodal hub ( k, i \in W_o, k \in H_o )</td>
</tr>
<tr>
<td>( A_{mij\phi} )</td>
<td>Geographical distance by road from intermodal hub ( m ) to warehouse ( j ) of region ( q, m \in H_d, j \in W_d )</td>
</tr>
<tr>
<td>( A_{km\phi} )</td>
<td>Geographical distance by rail from intermodal hub ( k ) to intermodal hub ( m \in H_o, m \in H_d )</td>
</tr>
<tr>
<td>( C_s )</td>
<td>Social cost conversion factor for emissions (Rs/tonne of CO2)</td>
</tr>
<tr>
<td>( C_{np} )</td>
<td>Social cost conversion factor for noise pollution in road travel (Rs/vehicle-km)</td>
</tr>
<tr>
<td>( C_{c} )</td>
<td>Social cost conversion factor for congestion in road travel (Rs/vehicle-km)</td>
</tr>
<tr>
<td>( C_{af} )</td>
<td>Social cost conversion factor for accidents in road travel (Rs/ton-km)</td>
</tr>
<tr>
<td>( C_{evf} )</td>
<td>Combined social cost conversion factor for negative externalities in rail travel (Rs/vehicle-km)</td>
</tr>
</tbody>
</table>

\[
\delta_{mt} \leq w_{mt}, \forall m, \forall t \quad (21)
\]

The problem is subjected to constraints defined in the Eq. (17) - (45). Eq. (17) links the shipment quantity allocation and route selection variables through a large number, \( M \) in such a way that there is positive movement quantity only if there exists a route in that direction. Eq. (18) and (19)
ensure that flow is routed through a hub only if it is open for origin and destination states respectively. Similarly, Eq. (20) and (21) enforce the restriction that only open origin or destination hubs are allowed to disrupt.

\[
y_{ikmj}^{\xi t} = \left[ \frac{x_{ikmj}^{\xi t}}{1 + x_{ikmj}^{\xi t}} \right], \forall i, \forall k, \forall m, \forall j, \forall \xi, \forall t
\]  

\[
\sum_{\xi \in \mathbb{Z}} y_{ikmj}^{\xi t} \leq 1, \forall i, \forall k, \forall m, \forall j, \forall t
\]  

\[
\sum_k z_{kt} = a, \forall t
\]  

\[
\sum_m w_{mt} = b, \forall t
\]  

\[
\sum_{\xi, i, k, m} x_{ikmj}^{\xi t} \geq D_{i j}, \forall j, \forall t
\]  

\[
\sum_{\xi, k, m, j} x_{ikmj}^{\xi t} \leq I_{i k}, \forall i, \forall t
\]  

\[
I_{i(i-1)} + P_{it} - \sum_{\xi, k, m, j} x_{ikmj}^{\xi t} = I_{i k}, \forall i, \forall t
\]  

\[
\sum_{\xi, i, m, j} x_{ikmj}^{\xi t} \leq U_k z_{kt}, \forall k, \forall t
\]  

\[
\sum_{\xi, i, k, j} x_{ikmj}^{\xi t} \leq U_m w_{mt}, \forall m, \forall t
\]  

The least integer function in Eq. (22) is defined to avoid the empty transport between origin and destination warehouses. The second constraint (Eq. (23)) ensures that single type of route condition is selected for a given route. Eq. (24) and (25), restrict the number of hubs located in origin and destination hubs in any time period to \( a \) and \( b \) respectively. The demand, available inventory and flow balance constraints are represented by Eq. (26), (27) and (28) respectively. The intermodal hub capacity restrictions for origin and destination hubs are enforced in Eq. (29) and (30).

\[
\sum_{\xi, k, m} (1 - \omega_\xi) x_{ikmj}^{\xi t} \leq \lambda_{it} V_{\phi}, \forall i, \forall t
\]  

\[
\sum_{\xi, i, k, j} (1 - \omega_\xi) x_{ikmj}^{\xi t} \leq \lambda_{it} V_{\phi}, \forall i, \forall t
\]
It is ensured that the food grain flow adheres to vehicle capacity restriction by Eq. (31), (32) and (33). A binary parameter, $\omega_i$ is explicitly defined to establish the difference between a hub node and a non-hub node in Eq. (31), where $\omega_i$ equals 1 if $i^{th}$ origin warehouse is a potential hub and equals 0 otherwise. The equation takes care of vehicle capacity restriction only at the origin non-hub nodes, whereas the Eq. (32) and (33) are written to ensure vehicle capacity restrictions at origin and destination hubs respectively.

\[
\sum_{\xi \in Z} y_{\text{iefj}}^{\xi} \geq z_{kt} + \gamma_{kt} + w_{mt} + \delta_{mt} - 3, \forall i, \forall k, \forall m, \forall j, \forall t
\]

(36)

After a hub is disrupted, there is a possibility that flow is routed through non-disrupted hubs. However, since the non-disrupted hubs are associated with limited intermodal handling and vehicle capacities, the possibility of traffic congestion and further flow disruption is large. Hence, in this paper, the demand of food grains after disruption is routed through the emergency hubs. They are equipped with high intermodal capacity and routing through them is associated with relatively higher cost of transportation. In this context, Eq. (34), (35) and (36) ensure that emergency route is selected respectively for the first, second and the third cases of disruption described earlier.

\[
z_{et} = 1, \forall t \in T
\]

(37)

\[
w_{ft} = 1, \forall t \in T
\]

(38)

\[
\gamma_{et} = 0, \forall t \in T
\]

(39)
\[ \delta_{ft} = 0, \forall t \in T \]  
\[ \sum_k \gamma_{kt} = n_o, \forall t \in T \]  
\[ \sum_m \delta_{mt} = n_d, \forall t \in T \]

The allocations made with respect to the emergency hubs, \( e \) and \( f \), in Eq. (37) and (38) ensure that they are always open. Constraints in Eq. (39) and (40) eliminate the possibility of emergency hubs being disrupted. Further, the number of disrupted hubs in the origin and destination hubs are restricted to \( n_o \) and \( n_d \) respectively by Eq. (41) and (42).

\[ x_{ikmj}^{\tilde{e}t} = 0, \forall i, \forall j, \forall t, \forall \tilde{\xi}, (i, k) \in H_o \]  
\[ x_{ikmj}^{\tilde{e}t} = 0, \forall i, \forall j, \forall \tilde{\xi}, \forall t, (j, m) \in H_d \]  
\[ x_{ikmj}^{\tilde{e}t} \geq 0, y_{ikmj}^{\tilde{e}t}, z_{ik}, \gamma_{ik}, w_{mj}, \delta_{mt} \in \{0,1\}, \forall i, \forall j, \forall k, \forall m, \forall \tilde{\xi}, \forall t \]

According to the hub and spoke topology, transportation occurs between the hubs of the origin and destination states but not in between the hubs of the same state. The Eq. (43) and (44) are written to ensure that there is no flow between two hubs of the same state. Finally, Eq. (45) represents non-negativity and integrality constraints for the given problem.

The above formulation is unique in the following aspects. First, the simultaneous minimization of transportation, hub location rerouting, environmental and social costs in the presence of disruption is novel to transportation modelling and particularly to food grain context. Second, along with the demand and flow conservation constraints, the vehicle capacity constraints for hub and non-hub nodes, constraints linking the different types of variables, default allocations, and emergency hub constraints, are engineered specifically to this context.
5. Solution approach

As discussed earlier, the extended version of p-hub median problem formulated in this paper poses to be NP-hard. Given that, for a particular instance of the problem, \( n_1, n_2, n_3, n_4, n_5 \) and \( n_6 \) denote the number of origin warehouses, origin hubs, destination hubs, destination warehouses, number of route conditions and time periods respectively, the general expressions for aggregated total number of variables (\( \Omega_1 \)) and constraints (\( \Omega_2 \)) for the problem instance are shown in Eq. (46) and (47).

\[
\Omega_1 = 2(n_1n_2n_3n_4n_5 + n_2 + n_3)n_6
\]

\[
\Omega_2 = n_6 \left[ 4n_1n_2n_3n_4n_5 + 2n_1n_2n_3n_4 + 3n_1 + 5n_2 + 5n_3 + n_4 + n_3n_4(n_2 - 1)(n_1 + n_2n_5) 
+ n_2(n_3 - 1)(n_3 + n_2n_5) + 8 \right]
\]

(46)

For each instance, the problem has to deal with \( O(n_1n_2n_3n_4n_5n_6) \) and \( O(n_6(n_1n_2n_3n_4n_5 + n_2n_3n_5(n_2n_4 + n_1n_5))) \) variables and constraints respectively. Furthermore, due to the 3-degree nonlinear objective function, few 2-degree constraints and a least integer function, the feasible region for the given problem is highly discrete. As a result, the nature of the objective function increasingly tends to become non-continuous, non-differentiable and non-convex, thus, classifying the problem to be of complex nature.

In the literature, such complex problems are often addressed by metaheuristic approaches, as it becomes computationally intractable by exact solution algorithms. Parvaresh et al. (2014) addressed the PHMI problem using simulated annealing and tabu search techniques. However, in these approaches, the solution evolves through a single point and significant amount of time is consumed to arrive at near optimal solutions. To overcome this concern, Azizi et al. (2016) proposed a genetic algorithm based approach to solve PHMI variants based on multiple start points. In recent times, many metaheuristics including genetic algorithm (GA), simulated
annealing (SA), chemical reaction optimization (CRO), and ant colony optimization (ACO) have been implemented (Asgari et al. 2013; Mogale et al. 2016) on food grain supply chain problems. Owing to the computational complexity of the proposed MINLP and NP hard nature of its own, in this paper, a superior variant of particle swarm optimization (PSO) algorithm known as particle swarm optimization with differential evolution (PSODE) (Epitropakis et al. 2012) is used to solve the problem.

Particle swarm optimization (Kennedy and Eberhart, 1995) and differential evolution (DE) algorithms have been individually employed on wide range of complex optimization problems. However, both the algorithms individually lack the dexterity to escape local optimums and conquer immature convergence. Hybridization perspectives exploiting the benefits from both the sides were presented by many authors (Liu et al. 2010; Thangaraj et al. 2011; Xin et al. 2011). Epitropakis et al. 2012 implemented the hybridization and proposed PSODE, a combined form of canonical PSO and DE algorithms. The algorithm exploits the benefits of differential evolution strategy in social and cognitive dimensions experienced by each particle in the swarm to direct the search process. Due to its special characteristics, PSODE has a fast convergence rate as compared to it’s hybrid counterparts. In this paper, the algorithm has been appropriately tailored and employed to tackle the proposed constrained non-linear optimization problem. The detailed description of PSO, DE and PSODE algorithms and its adaptation to the current context is elucidated with suitable diagrams in the following sub-sections.

5.1 Particle swarm optimization

Particle swarm optimization (PSO) was invented by Kennedy and Eberhart, (1995) motivated by the natural phenomenon of birds flocking and fish schooling in search of food. Members of these species reach to their food by socially interacting with their neighbors with the fittest member
guiding the entire swarm of population in the right direction. The solution information is encoded in the form of a particle in PSO. Each particle of the population evolves through its experience and reaches the best solution. The canonical form of PSO was designed for continuous variables and later adapted for discrete and mixed integer formulations. For a swarm of population size, $N$ and iteration size $T$, the process of evolution of a D-dimensional particle, $(x_{i,j}^t, v_{i,j}^t)$, where $i \in \{1, 2, 3, \ldots D\}, j \in \{1, 2, 3, \ldots N\}$, and $t \in \{1, 2, 3, \ldots T\}$ is guided by the position and velocity update rules as shown in Eq. (48) and (49).

\begin{align}
    v_{i,j}^{t+1} &= \omega v_{i,j}^t + c_1 r_1 (p_{best_{i,j}}^t - x_{i,j}^t) + c_2 r_2 (g_{best_j}^t - x_{i,j}^t) \quad (48) \\
    x_{i,j}^{t+1} &= x_{i,j}^t + v_{i,j}^{t+1} \quad (49)
\end{align}

In the Eq (48), $p_{best_{i,j}}$ and $g_{best_j}$ are best positions achieved by the particle and the entire swarm in the $t^{th}$ iteration. Here, $\omega$, $c_1$, $c_2$ are the inertia and acceleration coefficients, $r_1$ and $r_2$ are uniformly distributed random numbers in between $[0, 1]$. However, a remarkable disadvantage of this approach is the local entrapment of the search process due to poor exploration capability of the algorithm.

5.2 Differential evolution

Differential evolution (DE) algorithm is a stochastic global optimization technique developed to efficiently explore complex contours. It was developed by Storn and Price, (1996) to solve nonlinear and multimodal problems with computationally intensive cost functions. The algorithm uses weighted difference of the evolution between two randomly chosen individuals to modify the current state of solution. ‘Larger the better’ or ‘smaller the better’ greedy rule is used to select the survival individual appropriately in each iteration. The algorithm has fast convergence characteristics and better exploration capability. The mutation, crossover and selection operators
used in the algorithm guide the search process towards the global optimum. For a given individual 
\( X^*_i = \{ x^*_{i1}, x^*_{i2}, x^*_{i3}, \ldots, x^*_{in} \} \), such that, \( i \in \{1, 2, 3, \ldots, D\} \), the list of commonly used mutation strategies
to evaluate the mutated individual \( X'_i = \{ x'_{i1}, x'_{i2}, x'_{i3}, \ldots, x'_{in} \} \) are provided in Eq. (50)-(54), where,
\( r_1, r_2, r_3, r_4, r_5 \) are uniformly distributed random numbers in the range \([1, D]\). \( x'_{(best)j} \) is the best
known solution for \( t \) iterations.

\[
x'_i = x^*_i + \chi (x^*_i - x^n_{ij}) \tag{50}
\]

\[
x'_i = x^*_{(best)j} + \chi (x'^*_i - x^n_{ij}) \tag{51}
\]

\[
x'_i = x'_i + \chi (x'^*_{(best)j} - x'^*_i) + \chi (x'^*_{i1} - x'^*_{ij}) \tag{52}
\]

\[
x'_i = x'_{(best)j} + \chi (x'_i - x'_{ij}) + \chi (x'^*_i - x'^*_{ij}) \tag{53}
\]

\[
x'_i = x'^*_{ij} + \chi (x'_i - x'^*_{ij}) + \chi (x'^*_{ij} - x'^*_{ij}) \tag{54}
\]

Here, \( \chi (\chi \in [0, 2]) \) is an amplification factor defined according to Eq. (55), where, \( i_1, i_2, \) and \( i_3 \)
are uniformly distributed random numbers in between \([1, D]\) and \( i_1 \neq i_2 \neq i_3 \).

\[
F_i = F_i + N(0, 0.5)(F_{i2} - F_{i3}) \tag{55}
\]

The crossover operator in DE algorithm is guided by a crossover probability, \( CR \in [0,1] \). The
resulting individual, \( X''_i \in \{ x''_{i1}, x''_{i2}, x''_{i3}, \ldots, x''_{in} \} \) by the crossover between two individuals, \( X'_i \)
and \( X'_i \) are evaluated based on Eq. (56), where \( rand \) and \( j_{rand} \) are uniformly distributed random
numbers in between \([0, 1]\), and \([1, n]\) respectively. Finally, the selection of the survivor individual,
\( X''_{i+1} \), for a minimization problem is carried out by using the greedy rule at the end of each iteration
\( t \) as per Eq. (57), where \( f(X) \) is the fitness evaluation function defined for individual \( X \).
\[
x_{ij}^t = \begin{cases} 
    y_{ij}^t, & \text{if } rand \leq CR \text{ or } j = j_{rand} \\
    x_{ij}^t, & \text{otherwise}
\end{cases} 
\quad (56)
\]

\[
X_{i}^{t+1} = \begin{cases} 
    Z_{i}^t, & \text{if } f(Z_{i}^t) \leq f(X_{i}^t) \\
    X_{i}^t, & \text{otherwise}
\end{cases} 
\quad (57)
\]

5.3 Particle swarm optimization with differential evolution

Amalgamating the procedures of PSO and DE, a versatile variant of these two approaches was proposed by Liu et al. (2010) for constrained optimization problems and later generalized by Epitropakis et al. (2012). In this approach, the hybrid algorithm is initialized with two different equal sized populations, \( pop_1 \) and \( pop_2 \). The particle best from \( pop_1 \) is stored in \( pop_2 \). The members of \( pop_1 \) are sorted according to constraint violations in descending order and the members of \( pop_2 \) are mapped according to their particle best values. However, both the populations evolve separately, and the position and velocity of 50% population of \( pop_1 \) is updated according to PSO procedure. Violating individuals are redirected to the feasible region by using reflection operator as shown in Eq. (58)

\[
x_{ij}^t' = \begin{cases} 
    0.5(l(j) + x_{ij}^t), & \text{if } x_{ij}^t < l(j) \\
    0.5(u(j) + x_{ij}^t), & \text{if } x_{ij}^t > u(j) \\
    x_{ij}^t, & \text{otherwise}
\end{cases} 
\quad (58)
\]

Next, DE procedure is applied on \( pop_2 \). In this paper, each member of \( pop_2 \) generates three new offsprings based on mutation strategies shown in Eq. (50), (52) and (54). Here, these strategies are chosen in order obtain the maximum diversity in the resulting individuals. Boundary violations are repaired by treating the violated individuals with Eq. (59), where \( w_{ij}^t \) and \( w_{ij}^t' \) are violated and corrected offsprings respectively. Later, the selection procedure of DE is applied on the offsprings.
to update the particle best members of \( \text{pop}_2 \) at the end of iteration \( t \) according to Eq. (60), where \( G(X) \) function evaluates the constraint violations of candidate solution \( X \).

\[
\begin{align*}
 w'_i &= \begin{cases} 
 2l(j) - w'_j, & \text{if } w'_j < l(j) \\
 2u(j) - w'_j, & \text{if } w'_j > u(j) \\
 w'_j, & \text{otherwise}
\end{cases} \\
p_{\text{best}}^{t+1} &= \begin{cases} 
 W'_i, & \text{if } f(W'_i) < f(p_{\text{best}}^{t}) \cap G(W'_i) \leq G(p_{\text{best}}^{t}) \\
p_{\text{best}}^{t}, & \text{otherwise}
\end{cases}
\end{align*}
\]

(59) (60)

The process is continued in a loop until the maximum iteration size, \( E \), is reached. This method has a twofold advantage over the other known similar techniques. First, the approach requires less number of iterations to converge to better solutions as the particle best evolves rapidly towards the global best due to the efficient constraint violation and boundary condition handling schemes. Second, the hybrid approach easily escapes local entrapment by virtue of multiple mutation strategies which help to maintain adequate solution diversity in the random search process.

5.4 Adaption of PSODE

In this section, the adaption of PSODE to solve the proposed MINLP is described.

5.4.1 Particle encoding scheme

In swarm intelligence, particle encoding scheme defines the mapping between the problem and the algorithm. In the context of the current problem, the particle consists of an array of decision variables arranged according to the order, \( x_{ikmj}^{ij} \), \( y_{ikmj}^{ij} \), \( z_{kl} \), \( w_{mt} \), \( y_{kl} \), and \( \delta_{mt} \). For example, in an instance where \( n_1 = 6 \), \( n_2 = 3 \), \( n_3 = 5 \), \( n_4 = 9 \), \( n_5 = 2 \), and \( n_6 = 3 \) the total number of variables is 9768 (\( n_1, n_2, n_3, n_4, n_5, \) and \( n_6 \) hold the same meanings as defined at the start of section 5). The particle matrix is initialized with dimensions \([N,D]\), where \( N \) is the population size and \( D \) is the total number of decision variables in a problem instance. Fig. 3 illustrates the encoding scheme.
for this example showing the number and position of decision variables in a particle one after the other.

Fig. 3. Particle encoding scheme for problem instance (6,3,5,9,2,3)

5.4.2 Discretization

Discretization is a boundary condition handling technique used in stochastic search processes while dealing with mixed integer problems. After the evolution of a particle according to PSO and DE, the integer variables of the problem may belong to outside the feasible region or may be converted to continuous numbers. In such cases, the discretization process is used to enforce integrality constraints on the corresponding violated variables. Based on the nature of the integer variables, two types of discretization schemes are used: Integer variable discretization and Binary variable discretization. One should easily understand that binary variable discretization is a special case of integer variable discretization. In this paper, the binary variables $y_{ikmj}$, $z_{kt}$, $w_{mt}$, $\gamma_{kt}$, and $\delta_{mt}$ are treated with binary variable discretization scheme before updating the particle.

5.4.3 Pattern generation

Pattern generation is a procedure followed with respect to the variables and parameters while coding and decoding the particle encoding scheme. In a particle, each decision variable column is
associated with a number, $\phi_i$, where $i \in \{1, 2, 3, \ldots, D\}$. The number is used to recall the decision variable as and when required while dealing with the objective function and constraints.

Table 5. Sample decision variable pattern sequences for demand constraint (Eq. (19))

<table>
<thead>
<tr>
<th>Right hand side ($D_{jt}$)</th>
<th>$j$</th>
<th>$t$</th>
<th>Pattern for $x_{ikmj}^{st}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{11}$</td>
<td>1</td>
<td>1</td>
<td>1, 10, 19, ..., 802, ..., 1612</td>
</tr>
<tr>
<td>$D_{21}$</td>
<td>2</td>
<td>1</td>
<td>2, 11, 20, ..., 803, ..., 1613</td>
</tr>
<tr>
<td>$D_{31}$</td>
<td>3</td>
<td>1</td>
<td>3, 12, 21, ..., 804, ..., 1614</td>
</tr>
<tr>
<td>$D_{91}$</td>
<td>9</td>
<td>1</td>
<td>9, 18, 27, ..., 810, ..., 1620</td>
</tr>
<tr>
<td>$D_{12}$</td>
<td>1</td>
<td>2</td>
<td>1621, 1630, 1639, ..., 2422, ..., 3232</td>
</tr>
<tr>
<td>$D_{93}$</td>
<td>9</td>
<td>3</td>
<td>3249, 3258, 3267, ..., 4050, ..., 4860</td>
</tr>
</tbody>
</table>

A separate pattern for each expression that involves a decision variable is evaluated using $\phi_i$. For example, the pattern sequences for $x_{ikmj}^{st}$ in the demand constraint (Eq. (19)) for different combinations of $j$ and $t$ while solving medium size problem are as shown in Table 5. All the other expressions in the objective function and constraints are evaluated using similar mapping technique.

5.4.4 Fitness evaluation

The global fitness is the sum of objective function value and the constraint violation costs. The global fitness, $G(X)$, for a given solution vector $X$ is evaluated according to Eq. (57). Here, $V_n(X)$ is the degree of violation of $n^{th}$ constraint and $\pi_n$ is penalty for violating the constraint.

$$G(X) = TC + \sum_n \pi_n V_n(X)$$ (57)

Given that $H_i(X) \leq B_i$ and $H_m(X) = B_m$ are the set of inequality and equality constraints of the problem respectively, $V_n(X)$ is calculated for each of these cases differently as per Eq. (58)
\[ V_n(X) = \begin{cases} (H_l(X) - B_l)^+ & \text{if } n = l \\ (H_m(X) - B_m) & \text{if } n = m \end{cases} \]  

(58)

5.4.5 Flow diagram and pseudo code

The flow diagram for PSODE implementation is described in Fig. 4 and its pseudo code is provided later in Fig. 5.

![Flow diagram for PSODE implementation](image)

**Fig. 4.** Flow diagram for PSODE implementation
### Algorithm PSODE

| Initialize pop₁, n₁, n₂, n₃, n₄, n₅, n₆, c₁, c₂, r₁, r₂, r₃, r₄, r₅ |
|--------------------|----------------|----------------|----------------|
|                    |                |                |                |
| Set pop₂ = pop₁    |                |                |                |
| pop₂ = convert_to_feasible(pop₂) |                |                |                |
| Set iter = 0       |                |                |                |
| do while iter < Maximum iteration |
|     foreach member a ∈ first half of pop₁ |
|         calculate pbest, objective function value(f) and constraint penalties(G) |
|         update position and velocity using Eq. (44) and (45) |
|         if f(new pbest) + G(new pbest) < f(old pbest) + G(old pbest) |
|         pop₂(a) = new pbest |
|     end for |
|     Set gbest = gbest(pop₂) |
|     foreach member b ∈ pop₂ |
|         perform mutation, crossover and selection using Eq. (46), (48), (50), (52), and (53) |
|         pop₂ = convert_to_feasible(pop₂) |
|         update pbest using Eq. (56) |
|     end for |
|     Compare and update gbest |
|     iter = iter + 1 |
| end |

---

**Fig. 5.** Pseudocode for PSODE

### 6. Dataset and experiments

The mathematical model is initially validated on small dataset and later implemented for medium and large size datasets inspired from real geographical scenario of South India. The type of food grain considered for the experimental study is rice. The total number of variables in a dataset is considered as basis for determining the problem size. The rail and road unit transportation costs reflect the actual fares required for transporting the rice in between origin (Andhra Pradesh) and destination (Tamil Nadu) states. The real data collected through field survey comprised of monthly procurement, FCI warehouse storage capacities, and warehouse level food grain demand statistics for the financial year 2014-2015. The carbon tax prices for rail and road are adopted from reliable
online sources (http://www.iasparliament.com/current-affairs/carbon-tax-in-india). Due to scarcity in the availability of social cost data for Indian context, the social cost conversion factors are approximated based on previous studies in similar context by Reza et al. 2011. Information that was directly inaccessible such as intermodal capacity, fixed cost of locating a hub, unit rerouting costs and vehicle resource availability are hypothetically simulated to the actual scale from secondary sources. A clear description of problem sets is given in Table 6 and Table 7 gives the region wise warehouse distribution in origin and destination states.

Table 6. Problem set description

<table>
<thead>
<tr>
<th>Problem set</th>
<th>Origin regions</th>
<th>Destination regions</th>
<th>Configuration ((n_1, n_2, n_3, n_4, n_5, n_6))</th>
<th>Number of variables</th>
<th>Number of constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>3</td>
<td>3</td>
<td>((5, 3, 3, 5, 2, 2))</td>
<td>1824</td>
<td>6056</td>
</tr>
<tr>
<td>Medium</td>
<td>3</td>
<td>4</td>
<td>((6, 3, 5, 9, 2, 3))</td>
<td>9768</td>
<td>32733</td>
</tr>
<tr>
<td>Large</td>
<td>3</td>
<td>4</td>
<td>((10, 4, 4, 10, 3, 3))</td>
<td>28848</td>
<td>87624</td>
</tr>
</tbody>
</table>

Table 7. Region wise warehouse distribution

<table>
<thead>
<tr>
<th>Problem set</th>
<th>State type</th>
<th>Hub type</th>
<th>Region 1</th>
<th>Region 2</th>
<th>Region 3</th>
<th>Region 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>Origin</td>
<td>Hub</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non hub</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Destination</td>
<td>Hub</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non hub</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>Medium</td>
<td>Origin</td>
<td>Hub</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non hub</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Destination</td>
<td>Hub</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non hub</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Large</td>
<td>Origin</td>
<td>Hub</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non hub</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Destination</td>
<td>Hub</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non hub</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

Later, sensitivity analysis is performed to observe the effect of changing problem conditions on the nature and scale of variation in the transportation, hub location, rerouting and total shipment costs. The problem condition is varied by considering different combinations of factors as defined.
in Table 8. These factors are chosen to represent the four key dimensions of the problem: hub location decisions, hub disruption, intermodal operations, and vehicle capacities. The design of experiments in sensitivity test follows L27 orthogonal array with three levels of each factor. The description of factor levels is provided in Table 9. A hub disruption level equal to zero indicates there is no disruption. Further details on results obtained from sensitivity analysis are discussed in the subsequent section.

Table 8. Definition of factors for sensitivity analysis

<table>
<thead>
<tr>
<th>Factor</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hub location level (F1)</td>
<td>The total number of hubs located in origin and destination states</td>
</tr>
<tr>
<td>Hub disruption level (F2)</td>
<td>Total number of hubs disrupted in origin and destination states</td>
</tr>
<tr>
<td>Cost consolidation factor (F3)</td>
<td>Amount of reduction in unit transportation cost as a result of flow consolidation</td>
</tr>
<tr>
<td>Vehicle resource availability (F4)</td>
<td>Maximum arc capacity of the network</td>
</tr>
</tbody>
</table>

Table 9. Level-wise description of factors for sensitivity analysis

<table>
<thead>
<tr>
<th>Level</th>
<th>Hub location level (a+b)</th>
<th>Hub disruption level (n_o+n_d)</th>
<th>Cost Consolidation factor (\alpha)</th>
<th>Vehicle resource availability (MT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6 (3+3)</td>
<td>0</td>
<td>0.2</td>
<td>600000</td>
</tr>
<tr>
<td>2</td>
<td>7 (3+4)</td>
<td>1</td>
<td>0.5</td>
<td>900000</td>
</tr>
<tr>
<td>3</td>
<td>8 (3+5)</td>
<td>2</td>
<td>0.8</td>
<td>1200000</td>
</tr>
</tbody>
</table>

*MT - Metric ton

7. Result and discussion

PSODE was implemented in MATLAB and executed on Windows 8, 64-bit Operating System consisting of 8 GB RAM and Intel Core i7 1.8 GHz processor. Initially, the experiments were run on small size data sets with 5 origin and destination warehouses, 3 potential origin and destination intermodal hubs, 2 types of route conditions and 2 time periods, in which case, the total number of variables and constraints amounted to 1824 and 6056 respectively (Table 6). The algorithm was able to solve the problem instance within 175 sec (Table 10). However, to validate the
approach for larger instances, later, the problem size was increased to 15 and 20 warehouses including both origin and destination states. In the expanded problems (medium and large size), due to the explosion in number of possible linkages the number of variables and constraints was found to increase exponentially (Table 6). The computational times also witnessed a significant increase with increase in problem size for medium and large size datasets. However, given the complexity and size of the problem, the computational times are found to lie within reasonable limits (Table 10). The model and algorithm settings for the experiments on small, medium and large size problems are shown in Tables 11 and 12 respectively. Fig. 6 shows global fitness convergence characteristics for all the three instances. Subsequently, the decision variables including shipment quantity, route selection and hub location pertaining to the global optimal costs are evaluated. However, as the decision variable count is high for all cases, their values are not presented here. The emphasis of the discussion carried out in this section is more cost implicative.

Table 10. Summary of results for small, medium and large size instances using PSODE

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transportation cost (Rs)</td>
<td>221418719.9</td>
<td>868992296</td>
<td>1671722311</td>
</tr>
<tr>
<td>Hub location cost (Rs)</td>
<td>10600000</td>
<td>26400000</td>
<td>25800000</td>
</tr>
<tr>
<td>Rerouting cost (Rs)</td>
<td>924684585</td>
<td>1990627198</td>
<td>2350973797</td>
</tr>
<tr>
<td>Environmental cost (Rs)</td>
<td>927467.49</td>
<td>4235135.88</td>
<td>11832632.52</td>
</tr>
<tr>
<td>Social cost (Rs)</td>
<td>183147595.6</td>
<td>412798578</td>
<td>1284664921</td>
</tr>
<tr>
<td>Total cost (Rs)</td>
<td>1340778367</td>
<td>3303053208</td>
<td>5344993661</td>
</tr>
<tr>
<td>Average execution time (s)</td>
<td>174.2</td>
<td>1123.36</td>
<td>2964.84</td>
</tr>
</tbody>
</table>

Table 11. Model settings

<table>
<thead>
<tr>
<th>Instance</th>
<th>Hub location level ((a+b))</th>
<th>Cost Consolidation factor ((\alpha))</th>
<th>Hub disruption level ((n_o+n_d))</th>
<th>Vehicle resource availability (MT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>3+3=6</td>
<td>0.8</td>
<td>1+1=2</td>
<td>364500</td>
</tr>
<tr>
<td>Medium</td>
<td>3+5=8</td>
<td>0.8</td>
<td>1+1=2</td>
<td>600000</td>
</tr>
<tr>
<td>Large</td>
<td>4+4=8</td>
<td>0.8</td>
<td>1+1=2</td>
<td>1141500</td>
</tr>
</tbody>
</table>
Table 12. Algorithm settings

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( N )</th>
<th>( E )</th>
<th>( \omega )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>300</td>
<td>0.9</td>
<td>0.9</td>
<td>0.1</td>
<td>0.98</td>
</tr>
</tbody>
</table>

As mentioned earlier, the paper captures sustainability in a holistic purview by including environmental and social cost components as shown in Eq. (2). The total cost obtained is the summation of two broad conflicting categories: (1) Total shipment costs (TRC+HC+RC) and (2) Sustainability costs (EC+SC). The appropriate choice of route condition for a given route from origin to destination state is required to ensure balanced sustainable transportation. The pareto plot capturing the two conflicting categories on the coordinate plane are shown in Figs. 7(a), 7(b), 7(c) for each of the three instances. Fig. 7 presents the best three solution fronts obtained by non-dominated sorting of the updated local solution pool after 300 iterations of the single objective optimization problem. The solution points to the top left corner of the graphs represent economically sustainable food grain transportation at high cost of environmental and social compatibility owing to substantial selection of lesser quality of route condition. In contrast, the
bottom right points represent economically and socially viable solutions achievable at significantly higher total shipment costs. The possibility to explore multi-objective solutions by considering the two cost categories as separate objectives makes an interesting research question. The appropriate choice of tradeoff between the conflicting cost categories keeping the interests of the stakeholders is left to the decision makers. The focus of the current work is limited to explore the possibility of mathematically capturing the aforesaid cost components using single objective optimization approach.

![Fig. 7 (a). Top three ranks of pareto fronts obtained for small dataset](image)
Fig. 7 (b). Top three ranks of pareto fronts obtained for small dataset

Fig. 7 (c). Top three ranks of pareto fronts obtained for small dataset
Finally, sensitivity test is conducted on medium size datasets to develop explicit insights on the behavior of individual and total costs against the model parameters described earlier in section 6. Table 13 summarizes the results of transportation cost, hub location cost, rerouting cost, environmental cost, social cost and the total shipment cost for different combinations of the factors described in Tables 8 and 9. Observations reveal that as the level of disruption escalates to higher levels, the mean transportation cost is initially reduced by 5.4% at level 2 and further reduces by 2% at level 3 (Fig. 8). It is useful to recollect that transportation cost is actually the portion of the total shipment cost that represents the transportation through non-disrupted hubs. The reduction in the mean transportation cost at level 2 and level 3 disruptions is attributed to the rerouting of the disrupted consignment through the emergency hubs and corresponding increase in the rerouting costs. The effect of increasing hub disruption levels on the hub location cost is seen in Fig. 9. Here, the initial rise in the mean hub location cost by 5.6% at disruption level 2 is because of the need to select high capacity hubs to accommodate for the loss of demand as the disruption level increases. At disruption level 3, a significant portion of the food grains is routed through the emergency hubs. Hence, the need to select high capacity hubs is reduced and the mean hub location cost is reduced by 7%. A substantial increase in the mean rerouting cost is observed with increasing level of disruption (Fig. 10). The social and environmental costs exhibit similar behavior and are observed to increase exponentially owing to substantial increase in vehicle kilometers with increasing level of disruption. (Figs. 11 and 12). The total cost is increased by 14% and 40% as the disruption level rises from level 1 to level 2 and level 3 respectively (Fig 13).

By definition, increase in cost consolidation factor implies a reduction in the benefit of consolidation. The mean transportation and mean total costs are found to increase by 77% and 57% respectively as the benefit of consolidation is reduced from level 1 to level 2, whereas they
are increased by 30% and 27% respectively as the benefit is reduced from level 2 to level 3 (Figs. 8 and 13). Hence, it is observed that the rate of increase in the benefit of flow consolidation in terms of total cost is reduced as the benefit of consolidation increases from level 3 to level 2 and level 2 to level 1 respectively. According to Fig. 9, the hub location cost is almost unaffected as the cost consolidation factor is increased from level 2 to level 3, or, as the benefit of consolidation is increased from level 3 to level 2. The mean rerouting cost is found to significant increase (Fig 10) with increase in the cost consolidation factor from level 1 to level 2 and level 3. While the environmental cost exhibits random behavior with changing cost consolidation factor (Fig. 11), the social costs are found to proportionally increase with decreasing levels of consolidation due to the large number of vehicles used at lower levels of consolidation. (Fig. 12).

It is clear from Figs. 8-13 that the economic advantage of hub location level on the individual and total costs are visible at higher levels of the factor. There is a decrease in the mean transportation cost, mean environmental cost, mean social cost and mean total cost as the hub location level increases from level 2 to level 3 by approximately 8%, 4%, 2% and 6% respectively. The hub location cost is increased by 24% and 15% as the hub location level increases from level 1 to level 2 and level 3. The mean rerouting cost is observed to decrease by 36% and 4% for 2nd and 3rd levels of hub location. This decrease is attributed to decrease in the portion of total shipment shared by a single hub as more number of hubs are located and thus observing a lesser economic loss under disruption.

Finally, the increase in the level of vehicle resource availability has a positive effect on all the costs except for one case (Fig. 9). Unlike other cases, as the vehicle resource availability increases from level 1 to level 2, mean hub location cost is increased by 3.5%. This is because a higher capacity hub is selected given higher vehicle resources and the increase in hub location cost is
compensated by the significant decrease in the transportation and rerouting costs. The total costs are found to decrease approximately by 8% and 5% as the vehicle resource availability increases from level 1 to level 2 and level 3. The social and environmental cost are found to decrease at higher levels of vehicle resource availability because of the increasing potential of the transport to be accommodated through shorter distances.

Fig. 8. Effect of factors F1-F4 on mean transportation cost
Fig. 9. Effect of factors F1-F4 on mean hub location cost

Fig. 10. Effect of factors F1-F4 on mean rerouting cost
Fig. 11. Effect of factors F1-F4 on mean environmental cost

Fig. 12. Effect of factors F1-F4 on mean social cost
In a multi-level organization, the final outcome strictly aligns itself with the focus of the strategic decision makers and their choice of decision might belong to one of the following four categories. According to the first, if the agenda is restricted to minimizing solely the transportation cost, it is then intuitively favorable to maintain low hub location level, low cost consolidation factor and high vehicle resource availability. Fig. 8 validates the above conditions for this dataset and also reveals that the aforementioned objective is best achieved under situations of high level disruption. In the second category, if decision makers are aligned towards minimizing the hub location cost, then, results suggest that it is preferable to maintain low levels of hub location, medium level of cost consolidation factor, and higher vehicle resource availability (Fig. 9). Further, results indicate that this strategy is more applicable for conditions of high disruption. In the third category, where the management decision is inclined towards minimizing the rerouting costs, then, it is economical to maintain high level of hub location, low level of cost consolidation and medium level of vehicle resource availability (Fig. 10).
Table 13. Result summary of sensitivity test for experiments 1 to 27

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Level</th>
<th>Transportation Cost (Rs)</th>
<th>Hub Location Cost (Rs)</th>
<th>Rerouting Cost (Rs)</th>
<th>Environmental Cost (Rs)</th>
<th>Social Cost (Rs)</th>
<th>Total Cost (Rs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 1</td>
<td>678837716.2</td>
<td>17400000</td>
<td>0</td>
<td>1737340.56</td>
<td>203568798</td>
<td>901543854.3</td>
</tr>
<tr>
<td>2</td>
<td>1 1</td>
<td>691159827.7</td>
<td>17400000</td>
<td>0</td>
<td>1848451.67</td>
<td>191225583</td>
<td>901633862.2</td>
</tr>
<tr>
<td>3</td>
<td>1 1</td>
<td>566525392.9</td>
<td>17400000</td>
<td>0</td>
<td>1626118.05</td>
<td>215881344</td>
<td>801432855</td>
</tr>
<tr>
<td>4</td>
<td>2 2</td>
<td>944186091.7</td>
<td>18000000</td>
<td>257333732</td>
<td>2882311.64</td>
<td>309331107</td>
<td>1531733242</td>
</tr>
<tr>
<td>5</td>
<td>2 2</td>
<td>931864960.2</td>
<td>18000000</td>
<td>242172314</td>
<td>2671079.01</td>
<td>298207650</td>
<td>1492916004</td>
</tr>
<tr>
<td>6</td>
<td>2 2</td>
<td>957297203.5</td>
<td>18000000</td>
<td>275444843</td>
<td>2794623.99</td>
<td>311452320</td>
<td>1564988991</td>
</tr>
<tr>
<td>7</td>
<td>3 3</td>
<td>1155805561.2</td>
<td>17000000</td>
<td>1121274875</td>
<td>1926039.84</td>
<td>238080542</td>
<td>2534087019</td>
</tr>
<tr>
<td>8</td>
<td>3 3</td>
<td>104346215.6</td>
<td>17000000</td>
<td>1076918495</td>
<td>2037253.04</td>
<td>225737419</td>
<td>2365157382</td>
</tr>
<tr>
<td>9</td>
<td>3 3</td>
<td>1266920682.7</td>
<td>17000000</td>
<td>1155664945</td>
<td>1804727.18</td>
<td>249225666</td>
<td>2690616021</td>
</tr>
<tr>
<td>10</td>
<td>2 1</td>
<td>1019566197.8</td>
<td>19600000</td>
<td>0</td>
<td>1899646.92</td>
<td>247605366</td>
<td>1288671210</td>
</tr>
<tr>
<td>11</td>
<td>2 1</td>
<td>1000434782.4</td>
<td>19600000</td>
<td>0</td>
<td>1778325.15</td>
<td>231481909</td>
<td>1253295016</td>
</tr>
<tr>
<td>12</td>
<td>2 1</td>
<td>1136077309.2</td>
<td>19600000</td>
<td>0</td>
<td>2011168.87</td>
<td>260817687</td>
<td>1413106166</td>
</tr>
<tr>
<td>13</td>
<td>2 2</td>
<td>1480411192.4</td>
<td>23600000</td>
<td>345003226</td>
<td>2734464.6</td>
<td>340924885</td>
<td>2192673770</td>
</tr>
<tr>
<td>14</td>
<td>2 2</td>
<td>1324300079.3</td>
<td>23600000</td>
<td>326881314</td>
<td>2623143.1</td>
<td>321803672</td>
<td>1999208208</td>
</tr>
<tr>
<td>15</td>
<td>2 2</td>
<td>1591623303.8</td>
<td>23600000</td>
<td>358416216</td>
<td>2855811.9</td>
<td>353046096</td>
<td>2329541428</td>
</tr>
<tr>
<td>16</td>
<td>2 3</td>
<td>607005572.9</td>
<td>22000000</td>
<td>528683033</td>
<td>4523929.28</td>
<td>395702130</td>
<td>1557914665</td>
</tr>
<tr>
<td>17</td>
<td>2 3</td>
<td>620124384.4</td>
<td>22000000</td>
<td>499669555</td>
<td>4600472.73</td>
<td>403357006</td>
<td>1549751418</td>
</tr>
<tr>
<td>18</td>
<td>2 3</td>
<td>594728350.7</td>
<td>22000000</td>
<td>558626849</td>
<td>4747386.02</td>
<td>392825697</td>
<td>1572928283</td>
</tr>
<tr>
<td>19</td>
<td>3 1</td>
<td>1353804976.0</td>
<td>26400000</td>
<td>0</td>
<td>3968564.16</td>
<td>374036420</td>
<td>1758209960</td>
</tr>
<tr>
<td>20</td>
<td>3 1</td>
<td>1242237864.5</td>
<td>26400000</td>
<td>0</td>
<td>3857561.5</td>
<td>391159544</td>
<td>1663654970</td>
</tr>
<tr>
<td>21</td>
<td>3 1</td>
<td>1465072199.4</td>
<td>26400000</td>
<td>0</td>
<td>4069665.49</td>
<td>362865301</td>
<td>1858407166</td>
</tr>
<tr>
<td>22</td>
<td>3 2</td>
<td>473651432.8</td>
<td>25400000</td>
<td>67096031.6</td>
<td>2328619.8</td>
<td>224543941</td>
<td>793020252</td>
</tr>
<tr>
<td>23</td>
<td>3 2</td>
<td>484863554.3</td>
<td>25400000</td>
<td>65183682.7</td>
<td>2449965.25</td>
<td>212420817</td>
<td>790318019.5</td>
</tr>
<tr>
<td>24</td>
<td>3 2</td>
<td>461483621.2</td>
<td>25400000</td>
<td>68919949.7</td>
<td>2217277</td>
<td>236715755</td>
<td>794736603.3</td>
</tr>
<tr>
<td>25</td>
<td>3 2</td>
<td>1026046806.6</td>
<td>23200000</td>
<td>771030148</td>
<td>2569732.68</td>
<td>359825374</td>
<td>2182672061</td>
</tr>
<tr>
<td>26</td>
<td>3 2</td>
<td>1137170258.1</td>
<td>23200000</td>
<td>746315258</td>
<td>2458500.01</td>
<td>347668662</td>
<td>2256812678</td>
</tr>
<tr>
<td>27</td>
<td>3 2</td>
<td>1008925694.9</td>
<td>23200000</td>
<td>799420394</td>
<td>2720967.51</td>
<td>377936486</td>
<td>2212203543</td>
</tr>
</tbody>
</table>
In any case, lower the disruption level, higher is the benefit from this strategy. In the case where, the focus of the organization tends towards minimizing environmental and social costs, the lower levels of hub location, medium level of cost consolidation and higher levels of vehicle resource availability is much preferred. According to the holistic approach, where the management is willing to adopt an integrated sustainable strategy that would simultaneously minimize transportation, hub location, rerouting, environmental and social costs, our results suggest that high hub location and low cost consolidation and high vehicle resource availability are favorable (Fig. 13).

8. Conclusion and future work
The contributions of this research are three fold. First, it extends the present research in the domain of inter-modal transportation for the context of food grain supply chain, specifically under hub disruption. The paper addresses economic, environmental and social sustainability by incorporating rerouting costs, carbon emission costs and by quantifying negative externalities of emissions and freight transportation. Emergency hub restrictions are enforced to ensure the continuous flow of food grain shipments during disruptions. Further, it is marked by significant social relevance as it addresses staple food concerns with respect to Indian population. Thus, it strongly relates to economic, environmental and social dimensions of sustainability in the TBL perspective. Second, a mixed integer nonlinear optimization model is formulated that minimizes transportation, hub location, rerouting, environmental and social cost objectives in a multi-period setting for single food grain commodity. The resulting model is embedded in a hub and spoke system to satisfy food grain demand between origin and destination states. The proposed model accommodates for multiple route conditions and incorporates hub capacity, vehicle capacity and hub location restrictions in addition to demand and flow conservation constraints. Third, Particle
Swarm Optimization with Differential Evolution (PSODE) algorithm, a superior variant of traditional PSO, was tailored and employed to solve the proposed MINLP. The optimal combination of origin-destination (o-d) pair of warehouses and the respective shipment quantities are determined by incorporating decision variables $x_{ikmj}^t$ and $y_{ikmj}^t$. The decisions $z_{kt}$, and $w_{mt}$ are incorporated to evaluate allocation of origin and destination hubs respectively.

The model was tested on small, medium and large size datasets inspired from Indian food grain industry. PSODE was able to solve the small, medium and large instances within 174s, 1123s and 2965s respectively. Sensitivity analysis reveals that food grain demand is fulfilled with 14% increase in the mean total cost for single hub disruption case (level 1 disruption) and with 40% increase for multiple hub disruption (level 2 disruption). In addition to shipment quantity, route allocation and hub location decisions, the above formulation could also be helpful to the policy makers to determine the additional investment required by FCI to fulfill the demand through the emergency hubs during disruption. The distribution managers can be sensitized with respect to the variation in the costs as additional rakes and trucks are made available at the hub and non-hub nodes while fostering economic, ecological and social sustainability.

This study can be extended to other food grain commodities by incorporating perishability aspects, stochastic demand, uncertain travel times, service time constraints and other product specific attributes in the model. The benefits of cost consolidation is not limited to food grain context and may be implemented for geographically widespread manufacturing sectors. The social dimension can be further strengthened by quantifying customer and employee satisfaction levels. The applicability of the model may be widened to neighboring countries and other sectors with subtle changes in the model owing to variation in geographical territories and supply chain structure.
Acknowledgements

This research is conducted in the context of project titled, ‘Transportation management to implement NFSA (TMI)’ sponsored by Ministry of Human Resource and Development (MHRD), Department of Higher Education, Government of India [sanction letter No. F. NO. 4-25/2013-TS-I].

References


Bai, R., Wallace, S.W., Li, J., Chong, A.Y.L., 2014. Stochastic service network design with


