Minimal mean-square error for 3D MIMO beamforming weighting

C. Xu✉, J. Cosmas and Y. Zhang

The 3D MIMO beamforming system needs a weighting method to determine the direction of beam whist reducing the interference for other beam areas operating at the same carrier frequency. The challenge is to determine the weights of the 3D MIMO beams to direct each beam towards its cluster of user terminals while placing its nulls at undesired user directions to minimise undesired interference. Therefore, the signal-to-interference-plus-noise ratio should be increased while the interference from the side lobes of the other beams reduced. A weight determining method is presented that constructs horizontal and vertical array weights, respectively, by minimising the mean-square error between the array pattern vector and the unit vector, where the unit vector expresses the desired direction for the array pattern and zero vector expresses the undesired direction. Since the rectangular planar array can be viewed as M linear arrays of N elements, the weight of the M–Nth element can be obtained based on the horizontal and vertical array weights.

Introduction: The 3D MIMO beamforming is one of the key technologies for 5G system, which has a fundamental dichotomy. There are two considerations to solve. The first is how many users should be covered by one 3D beamforming, which will determine how many dipole antennas will be used to transmit the 3D MIMO signal. The K-means algorithm is a good scheme to solve this problem by grouping the user terminals into clusters that can be serviced by the 3D beamforming. The second is the range of signal-to-interference-plus-noise ratio (SINR) for the different user terminals in the main beam after the K-means algorithm has grouped the user terminals into different clusters.

K-means algorithm: The basic concept of clustering is to divide patterns into different groups (clusters). Amongst many different clustering algorithms, K-means algorithm is a better algorithm for clustering a low dimensional data set [1, 2]. In K-means algorithm, the user terminals are divided into several groups based on the vertical and horizontal angles between user terminals. If the angles between a user terminal and one cluster centre are the smallest among the angles to other cluster centres, then this user terminal will belong to the cluster corresponding to this cluster centre. For example, the cell will be separated into six areas and the user terminals are randomly distributed within each area. Then every area will be divided into two parts, which is based on the height of user terminals. Then the K-means algorithm is used to determine the cluster centre for each group. Fig. 1 shows one area that has six user terminals, which are covered by three beams.

There are three beams and 6 users in the fourth face for the ninth simulation

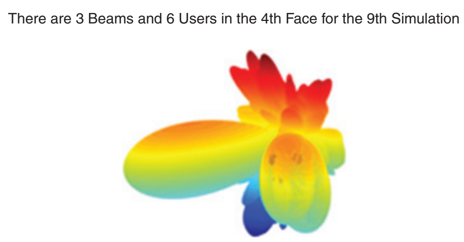

Fig. 1 *Three main beams for user terminals in one area*

In Fig. 1, the three main lobes have an effect on each other that is measured in SINR and the antenna array weights for parameterising them should be set to minimise the interference. In this Letter, the optimum antenna array weights are determined, which steers the beam towards its cluster of user terminals while maximising the SINR.

Determining the weight of each array element: For a rectangular array in the x–y plane, there are M elements in the x-direction and N elements in the y-direction. The x-directed elements are spaced dx apart and the y-directed elements are spaced dy apart. According to [3], we obtain the pattern of the entire $M \times N$ element array:

$$\mathrm{AF} = \mathrm{AF}_x \cdot \mathrm{AF}_y = \sum_{m=1}^{M} a_m e^{j(m-1)(kd_x \sin\theta\cos\varphi+\beta_x)} \sum_{n=1}^{N} b_n e^{j(n-1)(kd_y \sin\theta\sin\varphi+\beta_y)}$$

To allow the desired signal to be received without modification and reject the undesired interfering signals, let AF = 1 in the desired direction and AF = 0 in the undesired interfering direction. That is in the desired direction, we hope

$$\mathrm{AF}_x = 1 \text{ and } \mathrm{AF}_y = 1$$

in the undesired direction, we hope

$$\mathrm{AF}_x = 0 \text{ and } \mathrm{AF}_y = 0$$

i. Calculating the weight vector in the x-direction $\boldsymbol{W}_x$: Let $\boldsymbol{W}_x = [a_1 \quad a_2 \quad \dots \quad a_M]^{\mathrm{T}}$, then

$$\mathrm{AF}_x = \sum_{m=1}^{M} a_m e^{j(m-1)(kd_x \sin\theta\cos\varphi+\beta_x)} = [1 \quad e^{j(kd_x \sin\theta\cos\varphi+\beta_x)}a_2 \quad \cdots \quad e^{j(m-1)(kd_x \sin\theta\cos\varphi+\beta_x)}]\boldsymbol{W}_x \tag{1}$$

In our paper, if there are four beams in one area, then the one desired direction will be $\theta = \theta_0$ and $\varphi = \varphi_0$; three undesired directions are, respectively,

$$\theta = \theta_1 \text{ and } \varphi = \varphi_1$$
$$\theta = \theta_2 \text{ and } \varphi = \varphi_2$$
$$\theta = \theta_3 \text{ and } \varphi = \varphi_3$$

In the desired direction, from (1), we have

$$[1 \quad e^{j(kd_x \sin\theta_0 \cos\varphi_0+\beta_x)} \quad \dots \quad e^{j(m-1)(kd_x \sin\theta_0 \cos\varphi_{0=}+\beta_x)}]\boldsymbol{W}_x = 1 \tag{2}$$

In undesired direction, from (1), we have

$$[1 \quad e^{j(kd_x \sin\theta_1 \cos\varphi_1+\beta_x)}a_2 \quad \cdots \quad e^{j(m-1)(kd_x \sin\theta_1 \cos\varphi_{1=}+\beta_x)}]\boldsymbol{W}_x = 0 \tag{3}$$

$$[1 \quad e^{j(kd_x \sin\theta_2 \cos\varphi_2+\beta_x)}a_2 \quad \cdots \quad e^{j(m-1)(kd_x \sin\theta_2 \cos\varphi_2+\beta_x)}]\boldsymbol{W}_x = 0 \tag{4}$$

$$[1 \quad e^{j(kd_x \sin\theta_3 \cos\varphi_3+\beta_x)}a_2 \quad \cdots \quad e^{j(m-1)(kd_x \sin\theta_3 \cos\varphi_{32}+\beta_x)}]\boldsymbol{W}_x = 0 \tag{5}$$

According to (2)–(5), we obtain

$$\begin{bmatrix} 1 & e^{j(kd_x \sin\theta_0 \cos\varphi_0+\beta_x)} & \dots & e^{j(m-1)(kd_x \sin\theta_0 \cos\varphi_{0=}+\beta_x)} \\ 1 & e^{j(kd_x \sin\theta_1 \cos\varphi_1+\beta_x)} & \dots & e^{j(m-1)(kd_x \sin\theta_1 \cos\varphi_{1=}+\beta_x)} \\ 1 & e^{j(kd_x \sin\theta_2 \cos\varphi_2+\beta_x)} & \dots & e^{j(m-1)(kd_x \sin\theta_2 \cos\varphi_{2=}+\beta_x)} \\ 1 & e^{j(kd_x \sin\theta_3 \cos\varphi_3+\beta_x)} & \dots & e^{j(m-1)(kd_x \sin\theta_3 \cos\varphi_3+\beta_x)} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_M \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

If we denote

$$\boldsymbol{A} = \begin{bmatrix} 1 & e^{j(kd_x \sin\theta_0 \cos\varphi_0+\beta_x)} & \dots & e^{j(m-1)(kd_x \sin\theta_0 \cos\varphi_{0=}+\beta_x)} \\ 1 & e^{j(kd_x \sin\theta_1 \cos\varphi_1+\beta_x)} & \dots & e^{j(m-1)(kd_x \sin\theta_1 \cos\varphi_{1=}+\beta_x)} \\ 1 & e^{j(kd_x \sin\theta_2 \cos\varphi_2+\beta_x)} & \dots & e^{j(m-1)(kd_x \sin\theta_2 \cos\varphi_{2=}+\beta_x)} \\ 1 & e^{j(kd_x \sin\theta_3 \cos\varphi_3+\beta_x)} & \dots & e^{j(m-1)(kd_x \sin\theta_3 \cos\varphi_3+\beta_x)} \end{bmatrix}$$

and $b = [1 \quad 0 \quad \cdots \quad 0]^{\mathrm{T}}$, then $\boldsymbol{W}_x$ is one solution of the following equation we have:

$$\boldsymbol{A}X = b \tag{6}$$

When $\boldsymbol{A}$ is inversed, then $X = A^{-1}b$, however, $\boldsymbol{A}$ is $4 \times M$ matrix.

To obtain the weight vector in the x-direction $\boldsymbol{W}_x$, we need solve (6). Since $X = \mathrm{pinv}(\boldsymbol{A}) * b$ can minimise norm of vector $\boldsymbol{A}X - b$, we can take $X = \mathrm{pinv}(\boldsymbol{A}) * b$ as an approximate solution of (6), i.e.

$$\boldsymbol{W}_x = \mathrm{pinv}(\boldsymbol{A}) * b$$

where pinv ($\boldsymbol{A}$) is the Moore–Penrose pseudo inverse.

ii. In the same method, we can obtain the weight vector in the y-direction $\boldsymbol{W}_y$

$$\boldsymbol{W}_y = [b_1 \quad b_2 \quad \cdots \quad b_N]^{\mathrm{T}}$$

iii. Calculating the weight matrix $\boldsymbol{W}$

$$\boldsymbol{W} = \boldsymbol{W}_x(\boldsymbol{W}_y)^{\mathrm{T}} = (w_{mn})_{M\times N}$$

where w_{mn} is the weight of m–nth array element, and we have

$$w_{mn} = a_m b_n$$

Analysis and discussions on simulation results: For 3D MIMO the height of BS (base station) and UE will be taken into account during the simulation. The detailed simulation parameters are listed in Table 1.

Table 1: Simulation configuration parameters

Items	Parameters
The times of the simulation	20
Radius of base station (km) (d2D)	0.2–5
Height of the base station (m) (hBS)	25 or 150
Number of the antenna arrays	16 × 16
Number of antennas for beamforming	8 × 8
The size of antenna element	Width 0.06, length 0.06
Number of the move users	10
Height of the move user (m) (hUT)	1.5–25
Speed of motion	3 km/h
The system scenarios	3D-Uma
The carrier frequencies (GHz)	2
Subcarrier frequency spacing (MHz)	20
Thermal noise (dBm)	−174
Wavelength (cm)	11.99
The power of every antenna (dBm)	43
Base station	Gain 14 dBi, noise 5 dB
User terminal	Gain 0 dBi, noise 9 dB

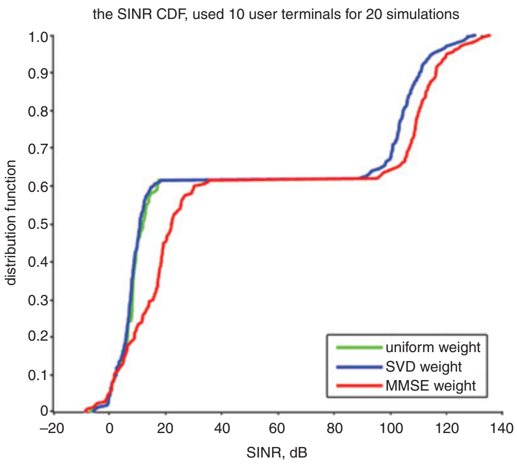

Fig. 2 *SINR CDF for three methods to determine the weight when high of BS is 25; green line shows that each array element has the uniform weight; blue line shows that the SVD algorithm is from the channel information; and red line shows the MMSE weight method*

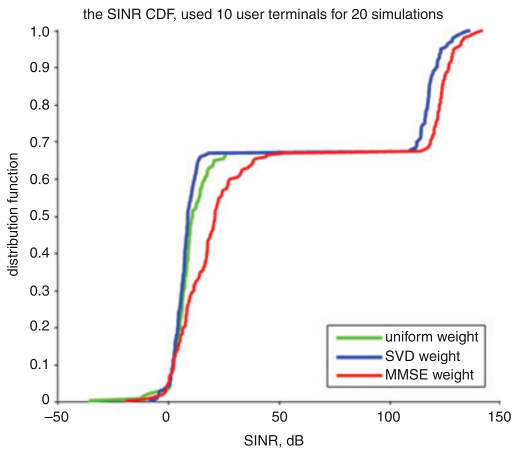

Fig. 3 *SINR CDF for three methods to determine the weight when high of BS is 150; green line shows that each array element has the uniform weight; blue line shows that the SVD algorithm is from the channel information; and red line shows the MMSE weight method*

In Figs. 2 and 3, the CDF (cumulative distribution function) of SINR is shown for 20 user terminals randomly distributed in one cell relative to the BS at heights 25 and 150 m, respectively.

To make a better comparison with the proposed weight method, there are three methods to determine the weight of array element in our simulation analysis. The first is set each array element has the uniform weight. The second is the singular value decomposition (SVD) algorithm from the channel information [4]. The third one is our proposed minimising the mean-square error (MMSE) method.

In Fig. 2, the most of values for MMSE method are larger than 10 dB in red line, which is about 80%. Moreover, the maximum value is more than 130 dB. The values in the MMSE weight line are better than the values in uniform weight line and SVD weight line. The proposed method has improved the SINR in 3D MIMO beamforming to clusters of user terminals.

In Fig. 3, 60% of the values in MMSE weight line are smaller than 30 dB, and corresponds with the performance observed in Fig. 2. Furthermore, the maximum value is more than 145 dB, which is larger than the maximum value in Fig. 2. The performance results presented in Figs. 2 and 3 are compared in Table 2.

Table 2: Results from Figs. 2 and 3

Different weight	SINR in Fig. 2 (25 m)		SINR in Fig. 3 (150 m)	
	CDF = 60%	CDF = 40%	CDF = 66%	CDF = 34%
Green line (dB)	<20	>90	<15	>110
Blue line (dB)	<20	>90	<25	>110
Red line(dB)	<40	>95	<50	>115

Table 2 shows that the SINR values for the BS at 150 m height are larger than the values for BS at 25 m height. Therefore, the SINR performance will be better when the height of BS is higher.

When the location of user terminal is almost parallel to BS, the SINR will increase because the main-lobes will overlap each other.

To sum up, the proposed weight method has decreased the interference between each beam and identified that the location of BS should be set at the top of high building for increasing performance.

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One or more of the Figures in this Letter are available in colour online.

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